X 410 "Business Applications of *Calculus*"

PROBLEM SET 1 [100 points]

PART I

As manager of a particular product line, you have data available for the past 11 sales periods. This data associates your product line's units sold "**x**" and total PROFIT "**P**" results for these sales periods.

Product	Red03
Units [**x**]	Profit [**P**]
10	**-33986**
20	**-31792**
100	**-9200**
130	**790**
190	**21418**
240	**37728**
300	**54000**
320	**58208**
380	**65840**
430	**65050**
500	**50000**

Section A: 1st Order Model

1. ***[4]*** Use Microsoft Excel's Chart feature to graph a plot of the data, assuming **P** = f(**x**). Add the most appropriate 1st order "trend line", the equation of this line, and the equation's *coefficient of determination*—its "[(R^2)]".

2. Answer the following questions using this 1st order model. Assume that, unless otherwise indicated, the restricted domain for "**x**" is **0** ≤ **x** ≤ **510** units.

   a. ***[4]*** Estimate Profit "**P**" @ "**x**" = **0** units and "**x**" = **70** units.

   b. ***[4]*** Estimate how many units "**x**" of the product must be sold in order to generate a PROFIT of $**0.00** and a PROFIT of $**35,000**.

   c. ***[4]*** Calculate how many product units "**x**" should be sold per sales period to *optimize* this product's PROFIT "**P**" and the value of "**P**" at this "**x**" value. Assume market constraints suggest the maximum number of product units that actually can be sold per sales period may not exceed…

      (1). …**510** (0 ≤ **x** ≤ 510 units). (2). …**300** (0 ≤ **x** ≤ 300 units).

   d. ***[4]*** Estimate *marginal* PROFIT "*m***P**" for this product if initially…

      (1). …**480** units were sold. (2). …**300** units were sold.

Section B: 2nd Order Model

1. ***[5]*** Use Microsoft Excel's Chart feature to graph a plot of the data, assuming **P** = *f*(**x**). Add the most appropriate 2nd order "trend line", the equation of this line, the equation's *coefficient of determination*—its "[(R^2)]"—and its adjusted *coefficient of determination*—its "[$(R^2)_{adj}$]".

2. Answer the following questions using this 2nd order model. Assume that, unless otherwise indicated, the restricted domain for "**x**" is **0** ≤ **x** ≤ **510** units.

   a. ***[4]*** Estimate Profit "**P**" @ "**x**" = **0** units and "**x**" = **70** units.

   b. ***[4]*** Estimate how many units "**x**" of the product must be sold in order to generate a PROFIT of $**0.00** and sold in a PROFIT of $**35,000**.

   c. ***[4]*** Calculate how many product units "**x**" should be sold per sales period to *optimize* this product's PROFIT "**P**" and the value of "**P**" at this "**x**" value. Assume market constraints suggest the maximum number of product units that actually can be sold per sales period may not exceed…

      (1). …**510** (0 ≤ **x** ≤ 510 units). (2). …**300** (0 ≤ **x** ≤ 300 units).

   d. ***[4]*** Use *differential calculus* to provide an estimate of *marginal* PROFIT "*m***P**" for this product if initially…

      (1). …**480** units were sold. (2). …**300** units were sold.

Section C: The Most Appropriate Model

1. ***[4]*** Identify which of the two PROFIT models derived above—1st or 2nd order—is most appropriate for estimating purposes, according to the "highest percent variation explained" criterion—a criterion based on [(R^2)] or [$(R^2)_{adj}$]. Based on which of the two models you feel is most appropriate, would you say that the results for the 1st order or 2nd order model are most realistic?

PART II

As manager of product line Blue03, you have the following data available for the past **6** sales periods. This data associates your product line's demand (units sold) "**x**" and unit price "**p**" results for these sales periods.

Product	Blue03
Demand [**x**]	Price [**p**]
200	**800**
400	**900**
600	**500**
1200	**600**
1600	**150**
2000	**50**

Section A: DEMAND Model Development

1. *1st Order:* Use the Chart feature of Microsoft Excel® to help derive…

   a. *[2]* …the product's "best fitting" 1st order model **p** = *f*(**x**).

   b. *[2]* …the model's *coefficient of determination* "[(R^2)]". Then, interpret the "[(R^2)]" value.

2. *2nd Order:* If the "[(R^2)]" value of the 1st order model is not "**+1**", use the Chart feature of Microsoft Excel® to help derive…

   a. *[2]* …the "best fitting" *polynomial*, 2nd order model **p** = *f*(**x**).

   b. *[3]* …identify the model's *coefficient of determination* "[(R^2)]", and compute its "adjusted" *coefficient of determination* "[$(R^2)_{adj}$]". Then, interpret the "[$(R^2)_{adj}$]" value.

Section B: Developing the Models to be Used in Subsequent Analyses

1. *[3]* **DEMAND**. Identify which of the DEMAND models derived above—1st order or 2nd order—best meets our course's "highest percent variation explained" criterion. Use this model to answer the questions that follow.

2. *[3]* **REVENUE**. Create the REVENUE model **R** = *f*(**x**) from the DEMAND model identified in "1" above.

3. *[3]* **COST**, **REVENUE** and **PROFIT**. Assume you had comparable COST "**C**" and units produced "**x**" data for the same 6 sales periods, and, after using Excel's Chart feature to develop 1st and 2nd order "trend line" equations and appropriate "[(R^2)]" values, you selected the 2nd order equation $\mathbf{C} = -0.1515(\mathbf{x})^2 + 345.01(\mathbf{x}) + 137559$ to use in further analyses. Create the PROFIT model $\mathbf{P} = f(\mathbf{x})$ from the COST model and from the REVENUE model identified in "2" above.

Section C: "Break Even", Optimization and Advanced Topics [$0 \le \mathbf{x} \le 1{,}100$ units]

1. *[3]* Calculate how many product units "**x**" must be produced and sold in order to generate a PROFIT of $**0.00**. Assume market constraints are currently such that "**x**" cannot exceed **1,100** units per sales period.

2. *[4]* Determine "**C**" and "**R**" at the quantity "**x**" where "**P**" = $**0.00**.

3. *Differential calculus* may be used as part of a process to develop optimization estimates for "$\mathbf{R}_{max}$" and "$\mathbf{P}_{max}$". Based on the market constraints shown below, calculate the number of product units "**x**" that should be sold per sales period to maximize REVENUE and PROFIT …then…calculate "$\mathbf{R}_{max}$" and "$\mathbf{P}_{max}$" at these "**x**" values.

   a. *[4]* **1,100** units ($0 \le \mathbf{x} \le 1{,}100$). b. *[4]* …**850** units ($0 \le \mathbf{x} \le 850$).

4. Determine the unit price "**p**" that should be charged per sales period to *optimize* this product's "**R**" and "**P**" based on the constraints of…

   a. *[4]* …3a above ($0 \le \mathbf{x} \le 1{,}100$ units). b. *[4]* …3b above ($0 \le \mathbf{x} \le 850$ units).

5. *[5]* Using your product line's **Cost**, **Revenue** and **Profit** models derived earlier, verify the following principle from economics: at the value of "**x**" (units produced and sold) where **Profit** "**P**" is a maximum, *marginal* **Cost** "*m***C**" = *marginal* **Revenue** "*m***R**".

6. Using *differential calculus* where necessary…

   a. *[3]* …find the value of the independent variable "**x**" associated with maximum *average* PROFIT "$a\mathbf{P}_{max}$" for this product line.

   b. *[3]* …develop the product line's *marginal* PROFIT ["$m\mathbf{P}$" or $(\mathbf{P})'$] expression.

   c. *[3]* …verify the assertion from econometrics that at the value of "**x**" associated with a product line's "$a\mathbf{P}_{max}$", *average* PROFIT and *marginal* PROFIT for this product line are equal.

Extra Credit (optional**)**

EC1. Corporate headquarters originally set your product line's PROFIT expectation for the next sales period at $**200,000**. Is this PROFIT expectation realistic? Support your answer quantitatively and/or graphically.

EC2. The "most appropriate" demand equation for a particular product is found to be $\mathbf{x} = 2{,}000 - 0.625(\mathbf{p})$. Develop this product's coefficient of *elasticity* expression and its Revenue equation $\mathbf{R} = f(\mathbf{p})$. Then, assuming there are no severe *domain* restrictions on price, determine the price where maximum Revenue occurs and the price associated with unit *elasticity* ($\eta = -1$). What do you observe about the two values?